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FINITE DEFORMATION EFFECTS IN PLASTICITY ANALYSIS*

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ABSTRACT

The non-linear kinematics of the combination of elastic and plastic deformations at finite strain provides the mathematical structure to examine aspects of elastic-plastic analysis more succinctly than is possible with the approach based on infinitesimal elastic strain.

Kinematic hardening represents the anisotropic component of strain hardening by a back stress $\underset{\sim}{\alpha}$. Application of current theory for finite deformation incorporates the effect of finite rotation by using the Jaumann derivative in the evolution equation for $\underset{\sim}{\alpha}$. This approach predicts oscillating shear stress for monotonically increasing simple shear strain but this anomaly can be eliminated by adopting a physically more meaningful modified Jaumann derivative.

1. INTRODUCTION

Structural metals can often be deformed to large strains without fracturing. The rational design of many engineering processes and structures demands stress and deformation analysis in the presence of finite strain. In order to anticipate and hence prevent the generation of cracks, high residual stresses or other forming defects in the manufacture of structural components, stress analysis must apply throughout a body and so elastic-plastic theory must be utilized. The neglect of the small elastic strain compared with plastic strain, and hence the adoption of rigid-plastic theory, prevents the determination of stresses in the rigid regions which may well comprise most of the work-piece in a metal forming process for example.

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The early development of elastic-plastic theory was based on infinitesimal deformation theory on the basis of which the total strain was equal to the sum of elastic and plastic components with a similar summation law applying also for strain rates. At finite strains there is a coupling between elastic and plastic deformation since plastic flow occurs in a material already stressed to yield, and hence subject to elastic strains, and these two components interact in the nonlinear kinematics of finite-deformation theory. In the currently commonly used approach to finite-deformation analysis, the summation of elastic and plastic strain rates to give the total strain rate is adopted. The significance of this assumption is examined in the light of the nonlinear kinematical theory. The latter gives a precision to the kinematics which permits many aspects of the theory to be investigated more succinctly.

2. ELASTIC-PLASTIC KINEMATICS

A body is considered in its initial undeformed unstressed configuration labelled $\underset{\sim}{X}$ in Fig. 1. After a deformation which

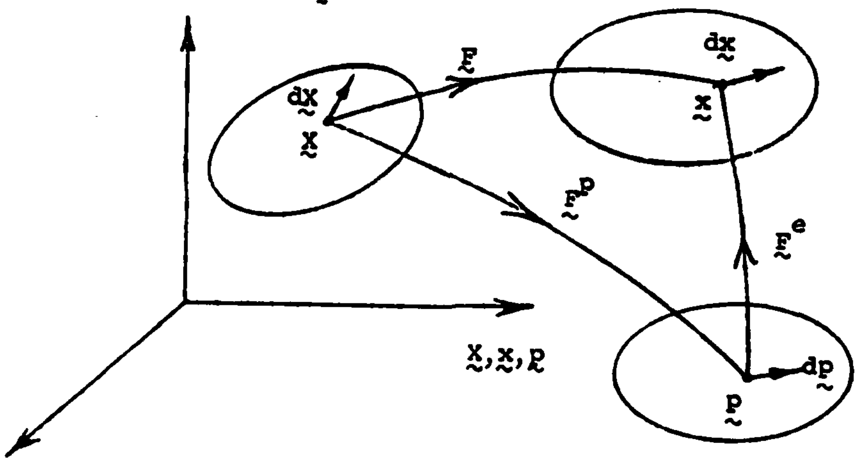

Fig. 1. Elastic-plastic deformation.

involves both elastic and plastic strain, the body occupies the configuration labelled $\underset{\sim}{x}$, the body having been deformed at time t according to the motion

$$\underset{\sim}{x} = \underset{\sim}{x}(\underset{\sim}{X}, t) \tag{2.1}$$

In order to uncouple the elastic and plastic deformations, the body is considered to be destressed from the configuration $\underset{\sim}{x}$ to the configuration $\underset{\sim}{p}$. It is assumed that the destressing to zero stress is purely elastic, although the theory can be modified to include materials which exhibit a strong Bauschinger effect involving plastic flow during the destressing to zero stress [1]. Since the stress is zero in the configuration $\underset{\sim}{p}$, the elastic strain is zero and hence $\underset{\sim}{p}$ exhibits the plastic strain. This is

also the plastic strain present in the configuration $\underset{\sim}{x}$ since only the elastic strain has changed during the destressing.

The elastic-plastic deformation which has taken place during the motion (2.1) is expressed by the deformation gradient $\underset{\sim}{F}$

$$\underset{\sim}{F} = \partial\underset{\sim}{x}/\partial\underset{\sim}{X} \quad (F_{ij} = \partial x_i/\partial X_j) \tag{2.2}$$

The mapping $\underset{\sim}{X} \rightarrow \underset{\sim}{p}$ with deformation gradient $\underset{\sim}{F}^p$ expresses the plastic deformation in both the configurations $\underset{\sim}{p}$ and $\underset{\sim}{x}$. The mapping $\underset{\sim}{p} \rightarrow \underset{\sim}{x}$ with the deformation gradient $\underset{\sim}{F}^e$ expresses the elastic deformation in the configuration $\underset{\sim}{x}$. Because removing the surface tractions from a body which has been subjected to non-homogeneous plastic strain leaves it in a state of residual stress, destressing involves considering the body sectioned into vanishingly small elements and thus the mapping to configuration $\underset{\sim}{p}$ from either $\underset{\sim}{X}$ or $\underset{\sim}{x}$ is not differentiable. $\underset{\sim}{F}^e$ and $\underset{\sim}{F}^p$ are then point functions of position but not deformation gradients. This causes no difficulty since the objective is to develop a constitutive relation for the stress in the configuration $\underset{\sim}{x}$ and the corresponding mapping from $\underset{\sim}{X}$ to $\underset{\sim}{x}$ is differentiable so that the deformation gradient $\underset{\sim}{F}$ does exist. Considering the body sectioned into small elements to achieve destressing to zero stress is analogous to machining away parts of a specimen to measure residual stresses. Such partition does not affect the basic elastic law so that using this in combination with measurements of change in strain, the stresses in the original whole component can be deduced. In practice, test specimens subjected to homogeneous strain are commonly used and then unloading the specimen does indeed destress it.

The configurations $\underset{\sim}{X}$, $\underset{\sim}{x}$ and $\underset{\sim}{p}$ can be obtained and measured experimentally, and the mapping $\underset{\sim}{X} \rightarrow \underset{\sim}{x}$ is identical geometrically with the resultant of the sequence $\underset{\sim}{X} \rightarrow \underset{\sim}{p} \rightarrow \underset{\sim}{x}$. The chain rule then generates the relation

$$\underset{\sim}{F} = \underset{\sim}{F}^e\underset{\sim}{F}^p \tag{2.3}$$

The generally non-commutative structure of this matrix product relation indicates that it is not compatible with the summation law for elastic and plastic strain or strain rate. It should perhaps be pointed out that the sequence of maps $\underset{\sim}{X} \rightarrow \underset{\sim}{p}(\underset{\sim}{X},t) \rightarrow \underset{\sim}{x}(\underset{\sim}{X},t)$ for varying time t cannot be carried out physically, because, for example, plastic flow cannot be generated at zero stress, but since the configurations can be achieved and then measured by passing through $\underset{\sim}{x}$, this does not affect the applicability of the relation (2.3).

The unstressed state $\underset{\sim}{p}$ is not unique since arbitrary rotation leaves it unstressed. Thus, without loss of generality, unstressing and hence the inverse, elastic deformation $\underset{\sim}{F}^e$, can be chosen to be pure deformation without rotation and thus can

be expressed by the symmetric deformation gradient matrix $\underset{\sim}{V}^e$. This choice simplifies some parts of the analysis.

Substituting (2.3) into the expression for total finite Lagrange strain

$$\underset{\sim}{E} = (\underset{\sim}{F}^T\underset{\sim}{F} - \underset{\sim}{I})/2 \tag{2.4}$$

(superscript T denoting transpose and $\underset{\sim}{I}$ the unit matrix) expresses the total strain in terms of the elastic and plastic strains calculated from $\underset{\sim}{F}^e$ and $\underset{\sim}{F}^p$ by relations analogous to (2.4):

$$\underset{\sim}{E} = \underset{\sim}{F}^{p^T} \underset{\sim}{E}^e \underset{\sim}{F}^p + \underset{\sim}{E}^p \tag{2.5}$$

For large plastic strains this is clearly far removed from summation of elastic and plastic strains.

In plasticity theory strain rate is appropriately defined in terms of the velocity field ($\underset{\sim}{v}(\underset{\sim}{x},t)$ in the current elastically-plastically deformed configuration $\underset{\sim}{x}$) as the symmetric part of the velocity gradient $\underset{\sim}{L}$:

$$\underset{\sim}{L} = \partial\underset{\sim}{v}/\partial\underset{\sim}{x} = (\partial\underset{\sim}{v}/\partial\underset{\sim}{X})(\partial\underset{\sim}{X}/\partial\underset{\sim}{x}) = \dot{\underset{\sim}{F}}\,\underset{\sim}{F}^{-1} \tag{2.6}$$

where $\dot{\underset{\sim}{F}}$ expresses a time derivative at fixed $\underset{\sim}{X}$, that is at a fixed material particle. Expressing $\underset{\sim}{L}$ as the sum of its symmetric and anti-symmetric parts

$$\underset{\sim}{L} = \underset{\sim}{D} + \underset{\sim}{W} \tag{2.7}$$

determines the deformation rate or stretching tensor $\underset{\sim}{D}$ and the spin $\underset{\sim}{W}$. The former expresses the rate of strain about the current configuration $\underset{\sim}{x}$ which is appropriate for plasticity analysis. Plasticity theory is commonly termed incremental or flow type, more akin to fluid behavior than to solid, the initial configuration $\underset{\sim}{X}$ playing a minor role unlike the case of elasticity theory.

Substituting (2.3) with $\underset{\sim}{V}^e$ into (2.6), incorporating (2.7), yields

$$\underset{\sim}{D} = \underset{\sim}{D}^e + \underset{\sim}{V}^e\underset{\sim}{D}^p\underset{\sim}{V}^{e^{-1}}\Big|_S + \underset{\sim}{V}^e\underset{\sim}{W}^p\underset{\sim}{V}^{e^{-1}}\Big|_S \tag{2.8}$$

where $\underset{\sim}{D}^e$, $\underset{\sim}{W}^e$, $\underset{\sim}{D}^p$ and $\underset{\sim}{W}^p$ are the symmetric and anti-symmetric parts of $\dot{V}^eV^{e^{-1}}$ and $\dot{F}^pF^{p^{-1}}$ respectively and subscripts S and A denote the symmetric and anti-symmetric parts respectively.

Now $\underset{\sim}{V}^e$ is the symmetric part of $(\underset{\sim}{I} + \partial u^e/\partial\underset{\sim}{p})$ where $\underset{\sim}{u}^e = \underset{\sim}{x} - \underset{\sim}{p}$ is the elastic displacement. Since elastic strains are usually of the order (yield stress divided by elastic modulus) $\sim 10^{-3}$,

$\underset{\sim}{V}^e$ is close to the unit matrix $\underset{\sim}{I}$. There are exceptions to this circumstance, for example in explosive generated shock waves in metals when volumetric elastic strain can be of the order unity. When $\underset{\sim}{V}^e \sim \underset{\sim}{I}$, (2.8) approximates the strain rate summation relation

$$\underset{\sim}{D} = \underset{\sim}{D}^e + \underset{\sim}{D}^p \tag{2.9}$$

since $\underset{\sim}{D}^p$ is symmetric and $\underset{\sim}{W}^p$ anti-symmetric. Thus the common assumption in finite-deformation elastic-plastic theory usually provides a good approximation to the theory based on nonlinear kinematics. However, in considering certain aspects of the structure of the theory, the precision of the nonlinear kinematical theory permits a more incisive investigation.

3. ELASTIC-PLASTIC CONSTITUTIVE RELATIONS

Plasticity exhibits a "flow" or "incremental" type of response to stress in that the plastic strain rate (or some characteristic of it) is determined by the stress variation, so that the plastic strain is determined by integrating the rate relations through the stress history. The final deformation thus depends on the stress history and similarly the stress depends on the deformation history. For a strain hardening material the strain-rate is given in terms of the current state (stress and hardening parameters) and the stress rate. For deformation of most structural metals at ambient temperatures, the plasticity law is of first order in time rates on each side of the constitutive equation so that any monotonically increasing function of time can replace the time variable without modifying the resulting stress and strain histories. For example a monotonically increasing displacement or strain variable would be appropriate. Materials which exhibit such a response are termed time-rate independent though they are of flow or incremental type. This paper is concerned with such materials, although the finite-deformation kinematics discussed applies equally well to rate dependent laws which usually apply at higher temperatures.

Since the plasticity law involves plastic strain rate, the formulation of elastic-plastic theory requires that the plasticity law and elasticity law be substituted into the total strain rate relation (2.8), or the approximation to it (2.9), to provide an expression for the resultant strain rate $\underset{\sim}{D}$ in terms of stress, stress rate and variables which express the influence of the previous history of the stress or deformation.

Since elasticity is usually expressed as a function relation for the stress in terms of the deformation, a rate form of this relation must be derived for substitution into (2.8) or (2.9). In the usual approach consideration is limited to small elastic strains and Hooke's law is used in the form

$$\varepsilon^e_{ij} = \frac{1+\nu}{E}\tau_{ij} - \frac{\nu}{E}\tau_{kk}\delta_{ij} \tag{3.1}$$

where ε is the infinitesimal strain and τ the Kirchhoff stress [2,3]. A rate form of this is commonly taken to be

$$D^e_{ij} = \frac{1+\nu}{E}\overset{\circ}{\tau}_{ij} - \frac{\nu}{E}\overset{\circ}{\tau}_{kk}\delta_{ij} \tag{3.2}$$

where $\overset{\circ}{\tau}$ is a time derivative of the stress tensor and D^e corresponds to the total rate of deformation tensor defined in (2.7) when the elastic velocity gradient is used.

Because the final constitutive relation must be valid for large plastic deformation, it must be so for large total strain and rotation. It is thus important to check that the constitutive relation is objective, i.e. that if a time-dependent rigid body motion is superimposed on the deformed configuration x, the effect on the stress at time t must be simply to rotate it according to the value of the superimposed rotation at that time. If (3.2) is substituted into (2.9) and combined with an objective plasticity law it is clear that the stress rate $\overset{\circ}{\tau}$ must transform under rigid body rotation expressed by the proper-orthogonal matrix Q (t) as does D

$$D \rightarrow Q\, D\, Q^T \tag{3.3}$$

There is an infinity of such rates to choose from [4], but the infinitesimal approximation built into (3.2) prevents a deductive choice. In current finite-element elastic-plastic programs two such rates have been adopted for finite-deformation analysis, the Jaumann rate and the Truesdell rate.

In the transformation of (3.1) to rate form (3.2), (3.1) was not formally differentiated because it is not a valid equality in the context of finite deformation theory. Rather a derivative of a type not uniquely selected was applied to the right-hand side, and the velocity field was introduced with which to express the "rate of strain" for the left-hand side. In contrast the nonlinear kinematical relation (2.8) can be combined with the finite-deformation-valid rubber elasticity law to permit formal differentiation of both sides of the elastic constitutive relation to provide a strain rate term which can be substituted into (2.8). This procedure was carried out in [5] using the Jaumann derivative. Since the same operator is applied to both sides of the elastic constitutive relation, an equality will result. The trick is to choose the operator so that the resulting strain-rate like term can be substituted into (2.8). It is necessary that the plasticity law will also mesh conveniently into (2.8). These components all meshed together in [5] to yield an elastic-plastic constitutive relation and a

corresponding variational principle. A simplification of the final result valid for small elastic strain yielded the conventional expression.

The plasticity analysis in [5] was based on isotropic hardening with Mises yield condition. The finite deformation elasticity law was also isotropic and not affected by plastic flow. In these circumstances the appropriate choice for the stress rate term has already been considered by Prager [4].

The plasticity law takes the form

$$D^p_{ij} = \frac{1}{h} \frac{\partial J_2}{\partial \tau_{ij}} \frac{\partial J_2}{\partial \tau_{mn}} \overset{\circ}{\tau}_{mn} \tag{3.4}$$

so that zero stress rate will imply zero plastic strain rate. And yet for the named objective stress rate definitions which Prager considered, other than the Jaumann rate, vanishing of the stress rate did not imply stationary behavior of the stress invariants. Thus for rate definitions other than the Jaumann rate, the stress invariants, and hence the yield condition, would change when no plastic flow was taking place. This is certainly not consistent with isotropic plasticity theory. Hence of the stress rate definitions considered, including the Truesdell rate, only the Jaumann rate is appropriate for elastic-plastic theory. Prager used the word "preferable" which seems to be too weak a statement.

Other aspects of elastic-plastic theory can be more precisely investigated using the nonlinear kinematics. For example the last term in (2.8) represents a rate of change of elastic strain which would appear as residual strain after a stress increment has been applied and then removed [6]. This fact was utilized in [5] by coupling it to $\underset{\sim}{D}^e$ to obtain a total elastic term which meshed conveniently with the elastic-plastic incremental constitutive relation development already described.

It was shown in [1] that this elastic strain rate, associated with the occurrence of the spin $\underset{\sim}{W}^p$ while the body is under stress, is parallel to the tangent to the yield surface in the nine-dimensional stress and strain space. This vector is thus normal to the stress vector and hence no power is expended by this strain rate. An analogous situation has a bearing on the investigation of the stability of plastic flow.

Analysis of the stability of plastic flow by Drucker [7], amplified by Palmer, Maier and Drucker [8] considers the power (rate of work) expended in plastic flow. This rate of work takes place in the configuration $\underset{\sim}{x}$ and is expressed, according to (2.8) by

$$\mathrm{tr}[\underset{\sim}{\tau}(\underset{\sim}{V}^e \underset{\sim}{D}^p \underset{\sim}{V}^{e^{-1}})_S] \tag{3.5}$$

where tr indicates the trace. Since τ is symmetric this is equal to

$$\mathrm{tr}(\tau V^{e} D^{p} V^{e^{-1}}) \qquad (3.6)$$

For the case under consideration associated with the Mises yield condition, the matrices τ, D^{p} and V^{e} all have the same principal directions so that the matrix products in (3.6) are commutative, hence (3.6) reduces to

$$\mathrm{tr}(\tau D^{p}) \qquad (3.7)$$

Thus the plastic rate of working by the stress on the motion in configuration x is the same as that for the purely plastically deformed configuration p. This uncouples the influence of elastic strain on the analysis of this question, a significant insight provided by the adoption of nonlinear finite deformation kinematics. Note that the commutativity of the deformation associated matrices in (3.6) determines the equality

$$V^{e} D^{p} V^{e^{-1}} = D^{p} \qquad (3.8)$$

so that normality of the plastic strain rate vector applies also to its manifestation in the x configuration. This result depends on the adoption of the Mises yield condition. For an isotropic yield condition involving the stress deviator invariant J_3 as well as J_2, for example the Tresca condition, D^{p} is not parallel to τ, nor hence to V^{e} and (3.8) is not valid.

4. STRESS ANALYSIS FOR KINEMATIC HARDENING

So far the discussion has been limited to isotropic hardening. However it is known that the anisotropic Bauschinger effect becomes more pronounced as the strain increases and can have a significant effect particularly if destressing occurs followed by reverse stressing. This can arise as the product emerges from a metal-forming process. It is therefore important to incorporate anisotropic hardening and the most utilized approach has been kinematic hardening or a combination of this with isotropic hardening. In the Workshop on Plasticity at Finite Deformation held at Stanford University in 1981, Nagtegaal and de Jong presented a paper [9] in which the stresses generated by simple shear to large deformation in materials which exhibit anisotropic hardening were evaluated using analysis currently accepted as valid for finite deformation. They obtained the unexpected result, for a material which strain hardens monotonically in tension, that the shear traction grows to a maximum value at a shear strain γ of the order unity and then oscillates with increasing strain. This paper has stimulated further study of the commonly accepted approach to the analysis of finite deformation in the presence of anisotropic hardening and a suggested modification of the theory follows.

In conformity with current practice for finite deformation analysis in the presence of kinematic hardening, [9] adopted an evolution equation for the back stress or shift tensor $\underset{\sim}{\alpha}$ (the current center of the yield surface) which relates the Jaumann derivative of $\underset{\sim}{\alpha}$ to the plastic strain rate. This incorporates effects of finite rotation and ensures objectivity of the evolution equation under superimposed rigid-body rotations.

Fig. 2 illustrates the kinematics of simple shear at finite strain. The velocity field is given by

$$v_1 = kx_2, \; v_2 = v_3 = 0 \qquad (4.1)$$

k being the rate of shear strain $\dot{\gamma}$. The material spin $\underset{\sim}{W}$ is uniform and constant and represents an angular velocity of magnitude k/2. The evolution equation takes the form

$$\overset{\circ}{\underset{\sim}{\alpha}} = C\underset{\sim}{D}^p \qquad (4.2)$$

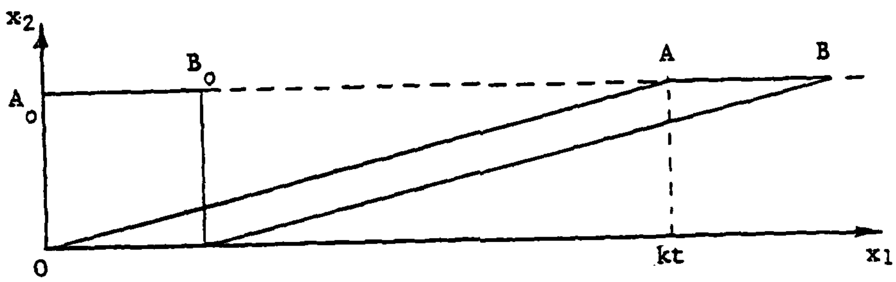

Fig. 2. Simple shear in the x_1 direction.

where C is a scalar function. Thus the material derivative which expresses the rate of change of $\underset{\sim}{\alpha}$ relative to the fixed coordinates shown in Fig. 2 is given by

$$\dot{\underset{\sim}{\alpha}} = C\underset{\sim}{D}^p + \underset{\sim}{W}\underset{\sim}{\alpha} - \underset{\sim}{\alpha}\underset{\sim}{W} \qquad (4.3)$$

It is pointed out in [10] that in view of the constant spin matrix $\underset{\sim}{W}$ the spin terms in (4.3) cause $\underset{\sim}{\alpha}$ to rotate continuously and that this causes the oscillations in stress already referred to.

It is shown in [10] and evident from Fig. 2 that no loci of material particles ever rotate through more than a limited angle as $t \to \infty$. For example the material particles lying initially on the x_2 axis OA_o only approach rotation through $\pi/2$ radians as $t \to \infty$. This poses a paradox since the back stress $\underset{\sim}{\alpha}$ is generated by micro-mechanisms embedded in the material and it seems implausible that $\underset{\sim}{\alpha}$ would rotate continuously if the physical source of its effects did not. The micro-mechanisms could be dislocations on crystallographic slip planes piled up against grain boundaries

or inclusions or residual stresses in crystallites generated by the inhomogeneous polycrystalline structure.

Since the back stress $\underset{\sim}{\alpha}$ expresses an average over the many active micro-mechanisms, it seems reasonable to select an average direction embedded in the material the rotation of which constitutes a contribution to the rotation of $\underset{\sim}{\alpha}$. A further contribution arises from the mechanisms being activated. The latter is represented by the first term on the right-hand side of (4.3). Study of the micro-mechanics of plastic flow in an anisotropic medium will be required to determine the orientation of the relevant material loci, but the macroscopic kinematic hardening law provides evidence from which this could be inferred. In [10] we suggested that the eigen-vector corresponding to the eigen-value of $\underset{\sim}{\alpha}$ of maximum absolute value provides a plausible choice. The rotation of the locus of material elements so oriented determines the spin $\underset{\sim}{W}^*$. A modified Jaumann derivative based on this spin is defined:

$$\overset{*}{\underset{\sim}{\alpha}} = \dot{\underset{\sim}{\alpha}} - \underset{\sim}{W}^*\underset{\sim}{\alpha} + \underset{\sim}{\alpha}\underset{\sim}{W}^* \tag{4.4}$$

and the corresponding evolution equation for $\underset{\sim}{\alpha}$ becomes

$$\overset{*}{\underset{\sim}{\alpha}} = C\underset{\sim}{D}^p \tag{4.5}$$

giving

$$\dot{\underset{\sim}{\alpha}} = C\underset{\sim}{D}^p + \underset{\sim}{W}^*\underset{\sim}{\alpha} - \underset{\sim}{\alpha}\underset{\sim}{W}^* \tag{4.6}$$

It is shown in [10] that the modified evolution equation (4.4) is objective.

In formulating the plastic flow relations care must be taken since the yield condition will be rotating with the tensor $\underset{\sim}{\alpha}$ and the effect of this must be eliminated from the definition of the stress rate in the expression for the plastic strain rate. This is treated in a manner analogous to that for the evolution equation by defining a modified Jaumann derivative of the stress based on the rotation of the $\underset{\sim}{\alpha}$ tensor. The summation of strain rates (2.9) was adopted since, as already discussed, it provides a satisfactory approximation.

Calculations were carried out [10] for kinematic hardening although the theory developed applies for combined kinematic-isotropic hardening. A model involving linear hardening in tension was assumed with constants chosen to approximate an aluminum alloy.

Fig. 3 shows the deduced stress variation based on rigid-plastic analysis. For this problem of homogeneous deformation rigid-plastic analysis is satisfactory since no rigid regions occur in the body. Since rigid-plastic theory involves incompressible deformation (which is consistent with the velocity boundary conditions) the stress can only be determined to with-

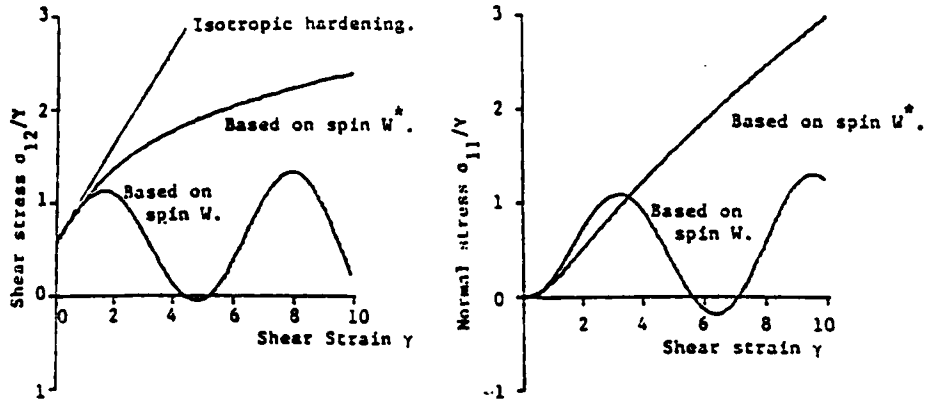

Fig. 3. Stress versus shear strain for rigid-plastic model.

in an arbitrary uniform hydrostatic stress. The normal stress plotted, σ_{11}, is in fact the stress-deviator. With boundary velocities prescribed, elastic-plastic analysis determines a unique stress field which is deviatoric and agrees with the rigid-plastic solution.

Fig. 3 shows that the method in current use based on the conventional Jaumann derivative predicts the stress oscillation in both shear and normal components obtained in [9]. Isotropic hardening with linear hardening in tension yields linear hardening in shear with no normal stresses generated. The new approach determines smooth hardening curves in shear and normal stress. No oscillations occur since now the hardening parameter $\underset{\sim}{\alpha}$ rotates towards a limiting position as t increases. The new approach gives close agreement with the other solutions at small strains and a gradual reduction of the shear hardening. The principal direction of the anisotropic hardening rotates towards the x_1 axis and this causes the reduced hardening in the shear curve.

Comparison of the analyses with the conventional and the modified Jaumann derivatives indicates that up to strains near 0.5 the difference in the solutions is small. For strains near 2 the difference reaches about 40% and grows rapidly with increasing strain. These results suggest that current codes for stress analysis at large strains which incorporate kinematic hardening should be reassessed if the possibility of serious errors in stress evaluations is to be avoided.

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REFERENCES

1. LEE, E.H. - Some Comments on Elastic-plastic Analysis. Int. J. Solids Structures, Vol 17, pp.859-872, 1981.

2. HILL, R. - Some Basic Principles in the Mechanics of Solids Without a Natural Time. J. Mech. Phys. Solids, Vol 7, pp.209-225, 1959.

3. McMEEKING, R.M. and RICE, J.R. - Finite-Element Formulations for Problems of Large Elastic-Plastic Deformation. Int. J. Solids Structures, Vol 11, pp.601-616, 1975.

4. PRAGER, W. - An Elementory Discussion of Definitions of Stress Rate, Quart. Appl. Math., Vol 18, pp.403-407, 1961.

5. LUBARDA, V.A. and LEE, E.H. - A Correct Definition of Elastic and Plastic Deformation and its Computational Significance. J. Appl. Mech, Vol 48, pp.35-40, 1981.

6. LEE, E.H. and McMEEKING, R.M. - Concerning Elastic and Plastic Components of Deformation. Int. J. Solids Structures, Vol 16, pp.715-721, 1980.

7. DRUCKER, D.C. - A More Fundamental Approach to Plastic Stress-Strain Relations. Proc. First U.S. Nat'l Congr. Appl. Mech., ASME, pp.487-491, 1951.

8. PALMER, A.C., MAIER, G. and DRUCKER, D.C. - Normality Relations and Convexity of Yield Surfaces for Unstable Materials or Structural Elements. J. Appl. Mech., Vol 34, pp.464-473, 1967.

9. NAGTEGAAL, J.C. and DE JONG, J.E. - Some Aspects of Non-isotropic Workhardening in Finite Deformation Plasticity, to appear in Proc. of the Workshop : Plasticity at Finite Deformation, Div. Appl. Mech., Stanford Univ., 1982.

10. LEE, E.H., MALLETT, R.L. and WERTHEIMER, T.B. - Stress Analysis for Kinematic Hardening in Finite-Deformation Plasticity. SUDAM Report No. 81-11, Div. of Applied Mechanics, Stanford University, 1981.

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The non-linear kinematics of the combination of elastic and plastic deformations at finite strain provides the mathematical structure to examine aspects of elastic-plastic analysis more succinctly than is possible with the approach based on infinitesimal elastic strain.

Kinematic hardening represents the anisotropic component of strain hardening by a back stress α. Application of current theory for finite deformation incorporates the effect of finite rotation by using the Jaumann derivative in the evolution

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20. equation for $\underset{\sim}{\alpha}$. This approach predicts oscillating shear stress for monotonically increasing simple shear strain but this anomaly can be eliminated by adopting a physically more meaningful modified Jaumann derivative.

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